**Example 1:**

Winco sells four types of products. The resources needed to produce one unit of each and the sales prices are given in the table below. Currently, 4,600 units of raw material and 5,000 labor hours are available. To meet customer demands, exactly 950 total units must be produced. Customers also demand that at least 400 units of product 4 be produced.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Resource | Product 1 | Product 2 | Product 3 | Product 4 |
| Raw Material | 2 | 3 | 4 | 7 |
| Hours of Labor | 3 | 4 | 5 | 6 |
| Sales Price ($) | 4 | 6 | 7 | 8 |

1. Clearly identify the decision variables and formulate an LP model to maximize sales revenue. Assume fractional units can be produced. *Hint: this LP only needs 4 variables.*

Variables: xi = amount of product i (where I = 1,2,3,4)

Maximize revenue = Z = 4x1+6x2+7x3+8x4

Constraints (Subject To) **dual variables**

X1+x2+x3 = 950 1 ~ unrestricted 1[Exactly 950 units]

X4 >= 400 2 <= 0 2[At least 400 units of product 4]

2x1+3x2+4x3+7x4 <= 4600 3 >= 0 3[raw materials]

3x1+4x2+5x3+6x4 <= 5000 4 >= 0 4[labor hours]

X1, x2, x3, x4 >= 0 [non-negativity]

(b) Write the general dual problem associated with the LP in Part (a). (DO NOT TRANSFORM OR REWRITE THE PRIMAL PROBLEM BEFORE WRITING THE GENERAL DUAL).

we will have 4 constraints in dual b/c we have 4 variables, and 4 variables in the dual b/c of 4 constraints in primal

minimize = 9501+ 4002+46003+50004

Subject to:

(Dual variable: x1 ≥ 0; thus constraint ≥)

(Dual variable: x2 >= 0; thus constraint >=)

(Dual variable: x3 >=0; thus constraint >=)

(Dual variable: x4 >=0; thus constraint >=)

1 ~ unrestricted

2 <= 0

3 >= 0

4 >= 0

**Example 2**

Write the general dual problem associated with the given LP.

*(Do not transform or rewrite the primal problem before writing the general dual)*

Minimize 20x1 + 6x2 – 5x3 + 4x4

Subject To Dual variables

4x1 + 2x2 + x3 – 3x4 ≤ –4 as it is a minimize problem

x1 – x2 – x3 + 2x4 ≥ 2

x1 + 5x3 + x4 = 0

x1 unrestricted

x2 ≤ 0

x3 ≤ 0

x4 ≥ 0

Dual

Variables: 3 variables

Constraints: 4 constraints

Maximize

Subject to:

(as x1 is unrestricted)

(x2 <=0 and the dual is a maximization problem)

(x3<=0, maximization constraint >=)

(x4>=0, maximization constraint <=)

“At least k out of m” and “Either-or constraints” Examples:

Example:

Consider the following 3 constraints:

What if I wanted 2 out of 3 or 1 out of 3? Lose convexity

Basic strategy: relax the constraints, and then force a subset of the constraints to be feasible:

how to write a constraint so that at least 1 constraint is satisfied/enforced?

if wanting 2 constraints satisfied.